S3: THERE ARE HIDDEN VARIABLES IN QUANTUM MECHANICS

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Abstract: The first two chapters in this article are dealing with the compatibility of Schrödinger equation with Maxwell equations. Although the reader may find them boring or difficult to understand, they are the continuation of my first two articles S1,S2 as mentioned in the introduction. Of course the essence of the title of this article lies within chapter 3, in which I discuss the fact that there are "hidden variables" in quantum mechanics, as Dr Albert Einstein first named them while he lived.

INTRODUCTION

In my previous articles S1)"An extension of Dirac notation and its consequences", S2)"An educative approach on the problem of defining magnetic permeability" I may have caused some confusion on the following matter: We have a magnetic induction made of two parts: $B=B_1+B_2$.

$$B^1 = C\nabla \times \nabla \phi \quad (1)$$

Since we know, and it is reasonable that the wave function psi is single valued, it must be a perfect differential and one may find after some elementary use of identities that:

$$\nabla \times \nabla \psi = 0 = \nabla \times \nabla\left(|\psi| e^{i\phi}\right) = i\psi\nabla \times \nabla \phi \quad (2)$$

Actually, the theorem of the combs informs us that there should be one more point that psi goes to zero as we change the principal quantum number. This should be before considering magnetic field, and in central potential (actually it is always central- but this is another big story) psi is made real through combinations of the proper degenerate (equal in energy) eigenfunctions. So, finally our magnetic field B^1 is different than zero only on specific surfaces.

CHAPTER 1

In my article “An extension of Dirac notation and its consequences” I found out that when taking the real part of functions in space of hermitian operators all vector algebra is obeyed. I also found out about the orthogonality of operators in that space.

All these directly imply (although I can not give a formal proof right now) that in transforming inside that space an operator to another we have equality. In that paper I found out that a term of magnetic work that should be zero was made of B_1M, where M is the magnetization .That was the beginning of my investigations. On these (equipotential) surfaces thus, it is that B_1 can be specified on.

HYPOTHESIS H1 Either: a) M=0, Or b)M is perpendicular to B_1 , and that should make B_1 have only a radial component.

The reader may refer now to a book of electromagnetism such as is the following:

“Foundations of Electromagnetic theory”
Reitz-Milford-Christy

Suppose now for a moment that we could make use of the steps found in this book. In the book psi is referred to as real. The only two “dangerous steps” taken there are a,b:

----------------- HYPOTHESIS H2 ------------

a)Suppose $\left(\nabla\times\nabla\psi\right)\times\vec{r}=0$,

b) $\nabla\cdot\vec{Z}=0\Rightarrow\vec{r}\cdot\nabla\times\nabla\psi=0$

If these two are satisfied we are lead to:

$$\left(\nabla\times\nabla\psi\right)=0 \quad \text{(i)}$$

Except for a spherical shell

------------- ------------- WHAT IT WOULD LEAD TO --------------------------------

One would then find out that if psi satisfies the equation:

$$\nabla^2\psi=-(E-U(r))\left(\frac{2m}{\hbar^2}\right)\psi \quad \text{(ii)}$$

If one were to follow the steps taken in that book, the vector considered would obey the same equation fro each spherical shell:

$$-i\hbar\nabla^2\left(\vec{r}\times\nabla\psi\right)=+i\hbar(E-U(r))\left(\frac{2m}{\hbar^2}\right)\vec{r}\times\nabla\psi \quad \text{(iii)}$$

This really translates to the fact that the components of angular momentum separately, such as L_{theta} , L_{phi} change the same way as psi does. This is referred as that the angular momentum operator has eigenvalues . Now, what happens on those shells is, that if nothing else occurred, the derivatives of psi as to the angles (on which angular momentum really depends) would also go to zero. This would make sense , since psi is zero everywhere on those shells. Also , it complies with hypothesis H1 since M=0 on those surfaces that B_1 exists. The evidence found is binding slowly.

---------- HOW THE STORY CONTINUES TAKING ----------------------------------

----- HYPOTHESIS H1 INVALID ON A SPHERICAL SHELL--------------------

We should now check the equation:

$$\nabla\times\nabla\times\left(\vec{r}\times\nabla\psi\right)+\vec{r}\times\nabla\left(\nabla\times\nabla\psi\right)=\frac{2i\hbar}{m}\left(E-U(r)\right)\vec{r}\times\nabla\psi$$

$$\left(\nabla^2\left(\vec{L}\right)-\left(\nabla\nabla\right)\cdot\vec{L}-i\hbar\vec{r}\times\left(\nabla\times\nabla\psi\right)\right)/(-i\hbar)= \qquad (3)$$

$$\frac{2i\hbar}{m}\left(E-U(r)\right)\vec{r}\times\nabla\psi$$

The two unwanted terms in eq. 3 are non zero only on the curve that is formed on the equipotential combic surfaces. As can be found by elementary arguments they are gradients as to the angles. If we do not wish Schrödinger equation to have unwanted limiting conditions (the only one imposed is psi tends to zero as we move to infinity) we should make them cancel each other. Let us see the unwanted terms:

$$\nabla\left(\nabla\cdot\vec{L}\right)-\nabla\left(\vec{r}\cdot\nabla\times\nabla\psi\right) \qquad (4)$$

Combining eq. 4 with eq.2 we find out , taking in mind of the fact that the terms proportional to psi should vanish the term reduces to:

$$Extra\_term=\nabla\psi\left(r\left(\nabla\times\nabla\psi\right)_r\right)=\left(r\frac{d\psi}{dr}\right)B_{1r}\hat{r} \qquad (5)$$

1) Proof that B_1 exists on the spherical sector of the shell:

Multiplying everything by psi* in equation 3 it must become identically zero. Now, the Schrödinger equation for the angular momentum components should be satisfied in a small volume around the delta function B_1. We integrate in the volume, and the term of eq.5 should vanish inside. After elementary manipulation and since B_1=B_1(omega) and psi^2=psi^2(r)psi(omega)^2 the term integrated over volume depends on the limit of the difference of |psi|^2(omega) B_{1r} (omega) , that vanishes.

2) The boundaries of the quantized flux:

There is something more however. A function of the form B_{1r}(r,omega) could exist on the bisection of psi(r)=0 and psi(theta,phi)=0, for there dpsi/dr=0. At the curves formed across the bisection of the cones psi(theta,phi)=0 and the spherical shell psi(r)=0 thus we have B_1r =B_1r(r, angles). On integrating over the volume there we should take zero possibility. It may be checked by the same method that the function dB^1/dr=delta(r) could exist along the curve considered and therefore these are the limits of the quantized flux. If we go to zero, certainly the phase may not be defined there as well, since we may multiply psi by any constant phase and get the same physical results, and also the angles are not defined perfectly there. This may be considered another combic point. Between the spherical sectors and the cone the flux is quantized and B_{ext} does not enter.

3) The flux is expelled from within the limits of the cone and the spherical sector.

There is no problem with the rest of components of B1. So, it is logical to assume through its proved continuity along these surfaces on which psi=0 within which it is enclosed that the total flux inside is zero inside there. The magnetic field is expelled from these regions, just like would be the case for a superconductor. The discontinuity of the derivatives of the phase in a path entering this region is satisfied, basic need to our reason of arguments. Inside that region either there is no magnetic field or it forms closed loops in which second case the current would flow outwards. We can not really say what happens inside right now, but this “box” certainly gives boundary conditions and is equivalent to some magnetized sphere added in a magnetic field. However no energy was consumed in creating this “expelling of flux”. It was simply absorbed by the “material”. The process itself is reversible.

CHAPTER 2

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = C\nabla \times \nabla \phi + \nabla \times \vec{A} \qquad (1)$$

That should clear some misunderstandings created in S1,S2.

The surfaces of psi=0 should separate the space into departments where flux is partially?(less than would be expected by the configuration of B_2) excluded and in parts in which the magnetic field is non zero. Along the intersection of these surfaces, by the way it is found that the angular momentum, the magnetic torque and the momentum can be perfectly defined. Through successive differentiations it is found that since there, dpsi/dr=Laplacian(psi)=0=psi=(E-U)psi ➔ Laplacian(L)=0=L ➔ Laplacian(Laplacian(psi)) =Laplacian(psi) and the story goes on and on, so the physical quantities can be determined exactly. It is to be expected, since only through discontinuity they can be defined so. Let us look at it in a more simple picture:

$$\Delta \psi \approx \nabla \cdot \vec{p} = 0 \qquad (2)$$

It comes from the fact that the wave does not spread and thus it is made of one wave length in the region. Since we do not expect them to be zero, (E-U)psi=0 ➔ E=U(r), no kinetic energy attached. It was also found that there is no charge on that region, and therefore:

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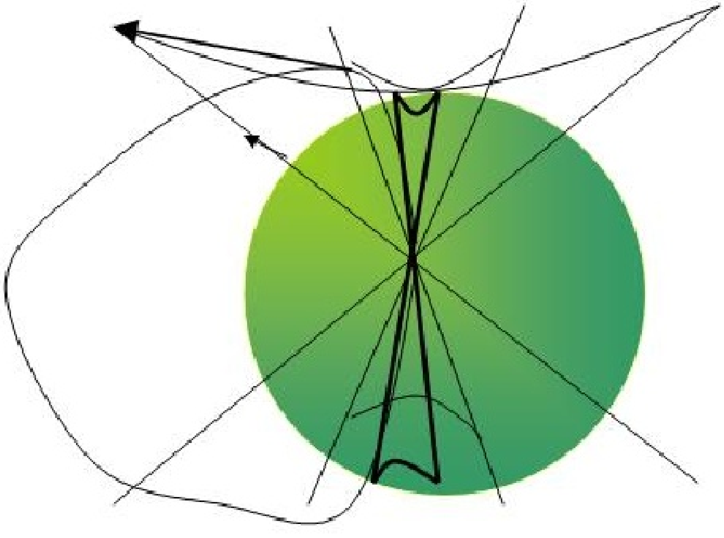
Continuation of the talk in chapter 1:

Most importantly, if we are given the boundary conditions for the equation found in S2, making the logical assumption that the permeabilities are positive:

$$\nabla^2 \vec{B} - C^2 \vec{B} = 0 \qquad (3)$$

The whole situation is quite the same with what happens in superconductors. After all, this was to be expected, since the current does not consume any energy. The flux is excluded continuously. It is a little unsure yet I refer the reader to any book about superconductors dealing with the London equations.

The reader may be quite familiar with the lobes of psi, surfaces of constant psi. These can only be formed however in the case of no magnetic field. There is no current in that case. If there was Ampere's law would not be valid. We know as well that for static magnetic fields, as there is no mixing of states, div(J)=d(|psi|)^2/dt=0. Except for these spherical shells we mentioned, on which there is no current, in all other space current density should form closed lines. The velocity being well defined on which J is dependent, should be perpendicular to those lines, as it is proportional to the gradient of psi. By imposing the boundary conditions in the previous chapter we have completely determined psi.



All these are illustrated in the above picture. The gradient of psi should look towards higher shells and different cones. It enters the small cones of psi=0. It is a special property of psi being a function having imaginary part that through being able to form only curves of constant value of psi , not only the density of the field lines of velocity may change but also the value of velocity along neighboring tubes of flux(of the same surface) That way, we are not able to decide the values of momentum everywhere from its values on a surface and we need the values of psi as well. The momentum In spite of the fact that div(velocity)≠0, the velocity field forms closed lines.

From the picture , one would expect that:

$$\nabla \cdot \vec{J} = 0 \Rightarrow \nabla |\psi|^2 \cdot \nabla \phi = 0$$

This also leads to:

$$\nabla \cdot \vec{J} = 0 \Rightarrow \Delta\phi = 0$$

$$\nabla \cdot \nabla \times \vec{B} = C\nabla \cdot \left(\vec{A} + \nabla\phi\right) = 0 \Rightarrow \nabla \cdot \vec{A} = -\Delta\phi = 0 = -\frac{\partial U}{\partial t}$$

The last is a consequence of Faraday's law and Ampere's law.

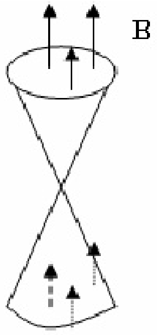
1) If there is no magnetic field applied, the wave function is real and the velocities imaginary. Since we measure real parts, there is no movement.

2) If a magnetic field is applied, and the particle is not in the basic state the velocities

If there is no magnetic field applied then the gradient of psi and the velocity (taking in mind the time dependence) vibrates between the combs of the wave function, This is compliable with the statistical nature of psi^2. It is made of subsystems, which change their phase space through time but conserving energy. Actually, this is the reason why we need to define both psi and dpsi/dx for boundary conditions in order to find the configuration, although we may know the potential U(r). In the classical picture, we have as Faraday imagined, tubes of flux , which by changing their surface proceed in a direction in the static case. I had mentioned in S2, that if div(J)=0 , closed lines should be formed.

The whole picture is not strange altogether. The equivalent of a "magnetic dipole" is formed when we apply a field. One could imagine that this magnetic dipole (it depends on the good quantum numbers m^l) is formed by current loops on those curves through the cones facing on opposite direction. The difference however is that they are perfect and their flux is quantized. The point is, energy has been consumed conservatively. I would only like to remind the reader, that the law for finding the magnetic field formed by a perfect current loop is B=gradient(omega) where omega is the solid angle formed by the observer and the loop surface.

Elementary magnetic dipole:



CHAPTER 3

Leaving for the moment the discussion on electromagnetism unfinished and with the principle of some uncertainty we now move to the essence of the whole story. To save some time I give to the reader one last result regarding electromagnetism:

Why did we chose magnetic permeability as being inversely proportional to the possibility $|psi|^2$? The answer will give us a good beginning to this chapter's discussion. The Lorentz force is not directly derived from Maxwell' s equations , but is crucial since it has to do with the change of reference systems. To talk about it let us first introduce the concept of force density:

$$\frac{d\vec{F}}{dV} = \frac{d}{d\vec{r}}\frac{d\vec{F}}{d\vec{S}} = \nabla P \qquad (1)$$

There is one certified case in which it can be applied, as can be read in books of electromagnetism as well as books of superconductivity:

$$\frac{d\vec{F}}{dV} = \vec{J} \times \vec{B} = \nabla P \qquad (2)$$

Let us follow the reasoning:

$$F = \int \vec{J} \times \vec{B} dV = \oint I d\vec{l} \times \vec{B} = I \oiint \nabla \times \vec{B} d\vec{S} = \mu I^2 \qquad (3)$$

That should say, the force is proportional to the kinetic energy of the currents, having a dependence on the gradient of self conduction factor L.

In our case this translates to:

$$\vec{J} \times \vec{B} = \frac{|\psi|^2}{N} \nabla U_L = \nabla P \qquad (4)$$

Where $U_L = eA^2$ the energy term added to the Hamiltonian. This can be found by considering the term mB as an operator acting on psi, the integral over volume equivalent to ½ integral(MB) dV . One has to use vector identities and when he comes to the following term, he should say it is zero as counting the change of a curl of a vector to the direction of the vector:

$$\left(\nabla\phi + \vec{A}\right) \cdot \nabla\left(\nabla \times \nabla\phi + \vec{A}\right) = 0 \quad (5)$$

We also make use of:

$$\nabla\left(\vec{A} \cdot \vec{A}\right) \approx \left(\nabla \times \vec{A}\right) \times \vec{A} + \left(\vec{A} \cdot \nabla\right)\vec{A} \quad (6)$$

The last part should vanish if we demand the momentum gained by the magnetic field to be constant along the path of the “particle”:

$$\frac{d\vec{A}}{dt} = \frac{\partial \vec{A}}{\partial t} + \left(\vec{A} \cdot \nabla\right)\vec{A} = 0 = 0 + 0 \quad (7)$$

If we deploy, by means of the Euler equation for streamlines of current density (created for fluids initially) and use eq.7 we get:

$$\begin{aligned}
\frac{d\vec{J}}{dt} &= \frac{\partial |\psi|^2}{\partial t}\left(\nabla\phi + \vec{A}\right) + \\
&+ \left(\left(|\psi|^2 \left(\vec{A} + \nabla\phi\right)\right) \cdot \nabla |\psi|^2\right)\left(\vec{A} + \nabla\phi\right) = \\
&= 0 + |\psi|^4 \begin{pmatrix} (\vec{A} \cdot \nabla)\vec{A} + (\nabla\phi \cdot \nabla)\nabla\phi + \\ + (\nabla\phi \cdot \nabla)\vec{A} + \left(\vec{A} \cdot \nabla\right)\nabla\phi \end{pmatrix} + \\
&+ \vec{J}^2 \cdot \nabla |\psi|^4
\end{aligned} \qquad (8)$$

First of all we notice the fourth power of |psi| used in the expansion of the free energy volume density for superconductors by Ginzburg-Landau.

$$\frac{d\vec{J}}{dt} = \frac{\partial |\psi|^2}{\partial t}\left(\nabla\phi + \vec{A}\right) +$$
$$+ |\psi|^4 \begin{pmatrix} (\vec{A}\cdot\nabla)\vec{A} + \nabla(\Delta\phi) + \left(\nabla^2\left(\phi\vec{A}\right) - \vec{A}\Delta\phi\right) + \\ + \left(\nabla(\vec{A}\cdot\nabla\phi) - \nabla\cdot\vec{A}\nabla\phi\right) \end{pmatrix} + \quad (9)$$
$$+ |\psi|^4 \,\nabla |\psi|^4 \left(\nabla\phi + \vec{A}\right)^2$$

Continuing we get, after some algebra:

$$\frac{d\vec{J}}{dt} = \frac{\partial |\psi|^2}{\partial t}\left(\nabla\phi + \vec{A}\right) +$$
$$+ |\psi|^4 \begin{pmatrix} 0 + 0 + \nabla^2(\phi\vec{A}) + \\ (\left(\nabla^2(\phi\vec{A}) - \phi\nabla\vec{A}\right) - 0) \end{pmatrix} + \qquad (10)$$
$$+ |\psi|^4 \,\nabla |\psi|^4 \;\nabla\phi + \vec{A}^{\,2} =$$
$$= |\psi|^4 \begin{Bmatrix} 2\nabla\times\nabla\times\left(\left(\phi\vec{A}\right) - \left(2\nabla\;\nabla\right)\cdot(\phi\vec{A})\right) - \phi\nabla\vec{A} \\ + \nabla |\psi|^4 \left(\nabla\phi + \vec{A}\right)^2 \end{Bmatrix}$$

Let us take a moment to see what could be happening right now.

Now, imposing a different condition than eq.4, and that is the partial time derivative as to the time of A should change, then we have a wave of one frequency and constant B thus (in space-not in time) thus, of any kind, one should make use of the common practice in those cases to express A as:

$$\vec{A} = \vec{r}\times\vec{B}2 = \vec{A}2$$
$$B_1 = \nabla\times\nabla\phi = \nabla\times\vec{A}_1 \qquad (11)$$
$$\nabla\times\nabla\times\left(\phi\vec{A}\right)$$
$$\nabla\times\nabla\times\left(\phi\vec{A}\right) = \vec{A}_1\times\vec{B}_2 + \vec{A}_2\times\vec{B}_1 \quad (12)$$
$$\nabla\cdot\left(\phi\vec{A}\right) = 0 = \phi\frac{\partial U}{\partial t} + \nabla\phi\cdot\vec{A} = \qquad (13)$$
$$= \nabla\phi\cdot\vec{A} + (E_2 - E_1)/\hbar U$$

We are now going to take a daring step in that we will assume

$$\vec{A}_1\cdot\vec{A}_2 = \nabla\phi\cdot\vec{A} = 0 \quad (14)$$

Due to eq. 11,10 then one finds that:

$$\frac{d\vec{J}}{dt} = \frac{\partial |\psi|^2}{\partial t}\left(\vec{A}_1 + \vec{A}_2\right) + \\ |\psi|^4 \left(\vec{A}_1 \times \vec{B}_2 + \vec{A}_2 \times \vec{B}_1 + + \nabla |\psi|^4 \left(\nabla\phi + \vec{A}\right)^2\right) \qquad (15)$$

The first two terms in equation 15 inside the parenthesis should be zero if we want the total field to obey a sensible equation (curl(B)externalB=0). It should read:

$$\frac{d\vec{J}}{dt} = -|\psi|^4 \frac{\partial \mu(\vec{r})}{\partial t}\left(\vec{A}_1 + \vec{A}_2\right) \\ |\psi|^4 \left(\nabla |\psi|^4 \left(\nabla\phi + \vec{A}\right)^2\right) \qquad (16)$$

It would be very sensible that it should vanish:

$$\frac{\partial \mu(\vec{r})}{\partial t} \vec{A}_{total} = -\left(\vec{J} \cdot \vec{A}_{total}\right) \nabla |\psi|^2 \quad (17)$$

Due to the formation of wave and to the fact that Ampere’s law as we defined it should be valid:

$$\vec{J} \cdot \vec{A}_{total} = -\nabla^2 \vec{A} \cdot \vec{A} = -C\vec{A}^2 \qquad (18)$$

This also implies and is compatible with the fact that the rest of terms besides the time derivative of E should vanish in curl(B) equation and then :

$$\vec{A}_{total} = \vec{A} + \nabla\phi \quad (19)$$

Or else as one could find:

$$\vec{J} = -\frac{\partial P}{\partial t} \quad (20)$$ --- If a wave is to be supported –

We find out then that the position vector the gard(phi) and the A (of the srcinal field) are all perpendicular to each other and curl(B) is along the spherical shell and , since B can not be central we have a wave formed along the spherical shell (and it could be sustained by the magnetization which forms such waves).

A s we shall find out later on, A^2 is the energy U_L gained by the magnetic field (and therefore the electromagnetic field). We will see that U_Ld|psi|^2 should express some sort of work (which on our conservative configuration considered is zero). So, there is a force that does not arise from pressure. And what does it do? Well, as eq.17 shows it could be that this force could be proportional to (mui)dA/dt

------------- TALK ON THE VIRIAL THEOREM-----------------

Now, in equation 4, if we were to pass |psi|^2/N to the other side, the second term should contain the potential of the “squares of the currents” that is roughly speaking, something like the kinetic energy- since classically I≈ v . By the way we have found the form of pressure exerted on the system by the magnetic field through these cones. The other pressure arising from the electric field is easy to find since |psi|^2=dq/dV:

$$\nabla P = \frac{|\psi|^2}{N}\nabla U \qquad (60)$$

And now we move on to more important:

The virial theorem:

It states that <Fr>=-2<Kinetic Energy> = - 2<K.E.>

Although it is stated in a confused quantical language we may have:

$$\left\langle \vec{F}\cdot\vec{r}\right\rangle = \int \vec{r}\cdot\frac{d\vec{F}}{dV}dV = -2\int|\psi|^2\,(E-U)dV \qquad (70)$$

Transforming 6:

$$\Delta P = \nabla\cdot\nabla P = \nabla\cdot\left(\frac{|\psi|^2}{N}\nabla U\right) = \frac{\nabla|\psi|^2}{N}\cdot\nabla U \qquad (80)$$

$$\frac{d(K.E.)}{dV} = \rho = \frac{|\psi|^2}{N}\left(E-U\right) = \rho = \frac{dq}{dV}(E-U) \qquad (90)$$

$$\Delta\rho = (E-U)\Delta\frac{|\psi|^2}{N} - 2\Delta P \qquad (100)$$

Now, eq.90 reads with virial theorem and 60 reads like:

$$-2dq(E-U) = \vec{r}d\vec{F} \qquad (110)$$

On the other hand as is evident:

$$\vec{F}d\vec{r} = -q\frac{dU}{d\vec{r}}d\vec{r} = -qdU \qquad (120)$$

Assuming $d(\vec{F}.\vec{r}) = 0$ (130) , virial theorem is valid,
One finds apart from some constants to be determined:

$$q(r) = q_0 e^{-\frac{U(r)}{E}}$$

As we saw where there was no charge at all , momentum (and angular momentum) could be perfectly defined.

That brings in mind something about the mass …